

# Browser-Native Colebrook–White Friction Factors: Residual-Checked Roots and Explicit Approximations

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Interactive calculator: <https://agentcalc.com/darcy-weisbach-friction-factor-calculator>

## Abstract

The Darcy friction factor used in pipe-pressure calculations is elementary in laminar flow and implicit in turbulent flow. A browser calculator that iterates the Colebrook–White equation by squaring its right-hand side can still return a number when that right-hand side is not positive. On a relative roughness of 4 at Reynolds number  $10^5$ , that loop stops after four iterations and reports a Darcy factor of 218.07, whose residual is 0.135. The same loop, asked for the commercial-steel example  $\text{Re} = 2 \times 10^5$  and  $\varepsilon/D = 4.5 \times 10^{-4}$ , is numerically close to the root but was being described on the page as  $f \approx 0.02$ . The root is 0.018560. Substituting 0.02 overstates the wall loss by 7.76%. We replace the loop with Newton’s method on  $g = 1/\sqrt{f}$ , which has a strictly monotone residual, no positive root once  $\varepsilon/D \geq 3.7$ , and a reported residual. On 572 turbulent states with  $5 \times 10^3 \leq \text{Re} \leq 10^8$  and  $0 \leq \varepsilon/D \leq 0.05$ , the iteration finishes in at most three steps and leaves a residual below  $10^{-12}$ . Against those roots, the largest relative errors are 2.994% for Swamee–Jain, 1.395% for Haaland, and 0.00314% for Serghides. A sharp switch from  $64/\text{Re}$  to Colebrook at  $\text{Re} = 2000$  is a different modeling choice: at the example roughness the two candidates already differ by 55.6% at  $\text{Re} = 1999$ . The calculator now shows both in the transitional band and does not call that band a Colebrook transition. Colebrook’s transition is the smooth-to-rough change at high Reynolds number.

## 1 Introduction

Head loss in a straight round pipe is still written with the Darcy–Weisbach relation, and the number that carries the fluid and the wall into that relation is the Darcy friction factor [1, 2]. In laminar flow the factor is Poiseuille’s  $64/\text{Re}$ . In turbulent flow the engineering standard is the implicit Colebrook–White equation, drawn by Moody as the diagram that replaced a shelf of earlier charts [2, 3]. The equation is not difficult, but it is easy to solve badly. It is scalar and smooth on its physical domain, and it is also undefined, as a positive friction factor, once the relative roughness is large enough that the logarithm has nothing left to be the reciprocal square root of.

That failure is what a fixed-point loop can hide. The usual teaching iteration inserts a guess on the right-hand side, squares the result, and stops when two successive guesses agree. Squaring removes a sign. If the right-hand side is negative, the iteration can still settle on a positive number and report that the change has become small. The number is then not a root. A page that also rounds a genuine root of 0.01856 to “about 0.02,” and that labels every Reynolds number above 2000 as turbulent Colebrook flow, mixes three different kinds of error: a non-solution, a rounding that moves the pressure drop by several percent, and a regime label Colebrook did not mean.

Colebrook’s 1939 title refers to the transition between the smooth-pipe and rough-pipe laws, not to the change from laminar to turbulent motion [3]. The second change is Reynolds’s subject. He showed that the motion of water in a pipe may be direct or sinuous according to the relation between viscosity, diameter, and velocity, and that the threshold is sensitive to how quiet the inlet is [4]. The brackets  $\text{Re} = 2000$  and  $\text{Re} = 4000$  used in engineering tables are conventions laid over that fact. They are not roots of the Colebrook equation.

This note documents the [Darcy friction-factor calculator](#) on AgentCalc after those distinctions were made visible in the result. The turbulent factor is a Newton root of an explicit residual. The residual, the iteration count, the Fanning factor, the fully rough asymptote, and three explicit approximations are returned with it. Between Reynolds numbers 2000 and 4000 the page reports both  $64/\text{Re}$  and the Colebrook root and does not choose. The benchmarks below are the check that this output is the equation it names.

## 2 Model

### 2.1 Darcy–Weisbach and the factor of four

For steady fully developed flow in a straight round pipe, the frictional pressure drop over a length  $L$  is

$$\Delta P = f \frac{L}{D} \frac{\rho v^2}{2}, \quad (1)$$

and the equivalent head loss is  $h_f = f(L/D) v^2/(2g)$ . Here  $D$  is the inside diameter,  $\rho$  the density,  $v$  the bulk velocity, and  $g$  gravitational acceleration. The Darcy factor  $f$  is dimensionless. The Fanning factor, defined from the wall shear, is  $f/4$ . Inserting a Fanning factor into (1) understates the loss by four. The calculator returns both numbers and names which is which. It does not evaluate (1) itself: length, density, and velocity are a different calculation, and minor losses from fittings are not part of  $f$ .

The history of (1) runs from Weisbach’s and Darcy’s nineteenth-century resistance formulas through Poiseuille and Hagen on laminar capillaries to the turbulent logarithmic laws [1, 5]. What a modern calculator owes that history is not a longer narrative. It is a refusal to mix the laminar law, the turbulent implicit law, and the Fanning normalization in one unlabeled float.

### 2.2 Laminar law

For a round pipe at Reynolds number  $\text{Re} = \rho v D / \mu = v D / \nu$ , the Hagen–Poiseuille balance gives

$$f = \frac{64}{\text{Re}}. \quad (2)$$

Roughness does not appear. The derivation assumes a straight circular section, a constant viscosity, and a parabolic profile that has forgotten the inlet. The calculator uses (2) as the reported factor when  $\text{Re} < 2000$ , and it still computes the Colebrook root at the same point so the size of the switch is visible. Reynolds’s experiments are the reason not to treat 2000 as a material constant: sinuous motion can be delayed by a quiet inlet and provoked by a disturbed one [4].

### 2.3 Colebrook–White

For turbulent flow in commercial pipe, Colebrook combined the smooth-pipe and rough-pipe logarithmic laws into one implicit expression [3, 6]. In the normalization of (1),

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{\varepsilon}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}} \right), \quad (3)$$

where  $\varepsilon$  is an equivalent sand-grain roughness. The two terms inside the logarithm are the roughness contribution and the viscous contribution. Their ratio says which regime of turbulence the state is in. If the viscous term dominates, the wall is hydraulically smooth and  $f$  still falls as  $\text{Re}$  rises. If the roughness term dominates,  $f$  approaches the fully rough asymptote

$$\frac{1}{\sqrt{f_\infty}} = -2 \log_{10} \left( \frac{\varepsilon}{3.7D} \right) \quad (4)$$

and no longer depends on Reynolds number. Equation (4) is Colebrook’s form of the Nikuradse–von Kármán rough-pipe law [3, 7]. Written out,  $-2 \log_{10}(\varepsilon/(3.7D)) = -2 \log_{10}(\varepsilon/D) + 2 \log_{10}(3.7)$ , and  $2 \log_{10}(3.7) = 1.1364$ . The classical rough law in terms of radius,  $1/\sqrt{f} = 2 \log_{10}(r/\varepsilon) + 1.74$ , becomes  $1/\sqrt{f} = 2 \log_{10}(D/\varepsilon) + 1.138$  after the substitution  $r = D/2$ , because  $1.74 - 2 \log_{10}(2) = 1.138$ . The constant 3.7 is that conversion, to the rounding Colebrook used.

The smooth limit  $\varepsilon = 0$  is Prandtl’s smooth-pipe law with a slightly different constant. Prandtl’s form is  $1/\sqrt{f} = 2 \log_{10}(\text{Re}\sqrt{f}) - 0.8$ . Colebrook’s smooth form is the same expression with 2.51 in place of  $10^{0.4} = 2.51189$ , since

$$-2 \log_{10} \left( \frac{2.51}{\text{Re}\sqrt{f}} \right) = 2 \log_{10}(\text{Re}\sqrt{f}) - 2 \log_{10}(2.51)$$

and  $2 \log_{10}(2.51) = 0.79935$  rather than 0.8. At  $\text{Re} = 10^5$  and  $\varepsilon/D = 0$  the Colebrook root is  $f = 0.017990$ , and the two smooth laws differ by  $6.53 \times 10^{-4}$  in  $1/\sqrt{f}$ . That is a change of constant, not a second physical model.

Moody adopted (3) as the curve for commercial pipe and plotted it across Reynolds number and relative roughness [2]. Nikuradse’s uniform sand-grain pipes have an inflection in the transitional-roughness range that commercial surfaces do not follow; Colebrook’s formula is an interpolation aimed at the commercial data, not a digitization of the sand-grain figure [3, 6, 7]. The calculator solves (3). It does not trace Nikuradse’s inflection.

### 2.4 Where a positive root exists

Set  $g = 1/\sqrt{f} > 0$  and  $\text{rr} = \varepsilon/D$ . The residual of (3) is

$$\phi(g) = g + 2 \log_{10} \left( \frac{\text{rr}}{3.7} + \frac{2.51 g}{\text{Re}} \right). \quad (5)$$

A root of  $\phi$  is a root of Colebrook, and the value of  $\phi$  is the natural acceptance test. Differentiating,

$$\phi'(g) = 1 + \frac{2}{\ln 10} \cdot \frac{2.51/\text{Re}}{\text{rr}/3.7 + 2.51 g/\text{Re}}. \quad (6)$$

Whenever the logarithm is defined,  $\phi'(g) > 1$ . The residual is therefore strictly increasing, and there is at most one root.

There is also a sharp criterion for existence. Both terms in the logarithm are nonnegative for  $g > 0$ ,  $\text{Re} > 0$ , and  $\text{rr} \geq 0$ , so the argument is at least  $\text{rr}/3.7$ . If  $\text{rr} \geq 3.7$ , that argument is at least 1 for every  $g > 0$ , the logarithm is nonnegative, and  $\phi(g) > 0$ . No positive  $g$  is a root. The boundary  $\text{rr} = 3.7$  would require  $g = 0$ , which is not a finite friction factor. If  $0 \leq \text{rr} < 3.7$ , then  $\phi(g) \rightarrow -\infty$  or to the negative number  $2 \log_{10}(\text{rr}/3.7)$  as  $g \rightarrow 0^+$ , while  $\phi(g) \rightarrow +\infty$  as  $g \rightarrow \infty$ . Strict increase gives exactly one positive root.

A relative roughness of 3.7 means a roughness height several times the diameter. That is not a circular pipe, and it is not a near miss that a solver should paper over. The previous browser loop did paper over it. It evaluated the right-hand side of (3), squared it, and accepted the square whenever the result was a positive finite number. A negative right-hand side squares to a positive  $f$ . At  $\text{Re} = 10^5$  and  $\varepsilon/D = 4$  the loop met its successive-difference tolerance of  $10^{-7}$  in four iterations and returned  $f = 218.069$ . The residual (5) at that value is 0.135. At  $\varepsilon/D = 3.7$  and the same Reynolds number the denominator vanishes on the fourth iteration, the update is infinite, and the page responded with the message it used for a blank Reynolds number. Neither outcome is a friction factor.

## 2.5 Newton's method

The update used now is Newton's method on (5),

$$g_{n+1} = g_n - \frac{\phi(g_n)}{\phi'(g_n)}, \quad (7)$$

started from Haaland's explicit guess (9). Because  $\phi' > 1$  wherever the residual is defined, the derivative in (7) does not vanish on the physical domain. The accepted value is the first  $g$  with  $|\phi(g)| < 10^{-12}$ , and the page prints that residual and the iteration count. If a trial step is non-positive or non-finite the step is replaced by  $g/2$ ; on every turbulent state in Section 4 the unmodified Newton step already meets the tolerance.

An exact rearrangement of (3) in the Lambert  $W$  function is known [8, 9]. Clamond's point, which we follow in spirit, is that a short, carefully tested iteration can return the root to near-rounding accuracy without calling a special function [9]. Newton on  $\phi$  has the additional property the calculator needs: the quantity used to stop is the quantity a reader can check, namely the residual of (3). We do not claim a new algorithm. We claim that the number on the page is a root, and that the cases with no root are reported as such.

## 2.6 Explicit approximations

A large literature replaces (3) with formulas that can be evaluated once [10–14]. The calculator reports three turbulent approximations beside the Newton root, each with its relative difference. They are comparisons, not alternative modes.

Swamee and Jain give

$$f = \frac{0.25}{\left[ \log_{10} \left( \frac{\varepsilon}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2}. \quad (8)$$

Haaland gives

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10} \left[ \left( \frac{\varepsilon}{3.7D} \right)^{1.11} + \frac{6.9}{\text{Re}} \right]. \quad (9)$$

Serghides uses three successive evaluations of the Colebrook right-hand side and one rational correction. With

$$\begin{aligned} A &= -2 \log_{10} \left( \frac{\varepsilon}{3.7D} + \frac{12}{\text{Re}} \right), \\ B &= -2 \log_{10} \left( \frac{\varepsilon}{3.7D} + \frac{2.51A}{\text{Re}} \right), \\ C &= -2 \log_{10} \left( \frac{\varepsilon}{3.7D} + \frac{2.51B}{\text{Re}} \right), \end{aligned}$$

the approximation is

$$\frac{1}{\sqrt{f}} = A - \frac{(B - A)^2}{C - 2B + A}. \quad (10)$$

Brkić’s review is the map of this literature and of the error tables authors have published for it [14]. The percentages in Section 4 are not those tables. They are the relative differences between (8)–(10) and the Newton root of (5) on one stated grid.

Churchill’s 1977 expression is a different object [15]. It is an all-regime blend, constructed so that a single formula passes from the laminar wall-stress law through the transitional band into the turbulent logarithmic laws. The friction factor in that article is one eighth of the Darcy factor. The Darcy transcription used here multiplies the blended expression by eight,

$$f = 8 \left[ \left( \frac{8}{\text{Re}} \right)^{12} + (A + B)^{-3/2} \right]^{1/12}, \quad (11)$$

with  $A = (2.457 \ln(1/((7/\text{Re})^{0.9} + 0.27 \varepsilon/D)))^{16}$  and  $B = (37530/\text{Re})^{16}$ . Its laminar limit is (2). It is reported on the page, and it is not scored as a failed Colebrook approximation when  $\text{Re}$  is near 2000. In that band it is doing what it was built to do.

## 2.7 Two transitions

The word “transition” has two uses that the calculator now keeps apart. The Reynolds-number transition is the band  $2000 \leq \text{Re} < 4000$ , in which a circular pipe may be laminar, intermittent, or turbulent depending on the inlet [4]. The calculator reports both (2) and the Colebrook root there and names the band transitional. The Colebrook transition is the change, at high Reynolds number, from the viscous term in (3) to the roughness term, ending at (4) [3]. A state can be turbulent and still far from fully rough. The worked example below is that state.

## 3 Browser implementation

### 3.1 What a completed solve returns

The inputs are  $\text{Re} > 0$  and  $\varepsilon/D \geq 0$ . The defaults are the worked example,  $\text{Re} = 2 \times 10^5$  and  $\varepsilon/D = 4.5 \times 10^{-4}$ , which is commercial steel of roughness 0.045 mm in a pipe of inside diameter 0.10 m. The form accepts any positive real Reynolds number; an integer step attribute had been rejecting values such as 1999.5 before the solver saw them.

The reported Darcy factor is (2) below  $\text{Re} = 2000$ , the Colebrook root at  $\text{Re} \geq 4000$ , and neither one alone in between. The result always includes, when the root exists, the residual  $\phi$ , the iteration count, the Fanning factor, the fully rough asymptote (4) when  $\varepsilon > 0$ , the ratio of the

Table 1: Darcy factor at  $\text{Re} = 2 \times 10^5$  and  $\varepsilon/D = 4.5 \times 10^{-4}$ . Relative differences are measured from the Newton root of (5). The legacy row is the previous successive-difference loop, retained as a benchmark and removed from the page.

| Evaluation                 | $f$        | Relative to Newton               |
|----------------------------|------------|----------------------------------|
| Newton on $g = 1/\sqrt{f}$ | 0.01856015 | residual $-7.11 \times 10^{-15}$ |
| Legacy fixed point         | 0.01856015 | residual $9.83 \times 10^{-8}$   |
| Swamee–Jain (8)            | 0.01866521 | +0.566%                          |
| Haaland (9)                | 0.01836974 | −1.026%                          |
| Serghides (10)             | 0.01856014 | $-3.92 \times 10^{-5}\%$         |
| Churchill (11)             | 0.01866642 | +0.573%                          |
| Fully rough (4)            | 0.01631094 | asymptote, not a root            |
| 64/Re                      | 0.00032000 | wrong regime                     |

viscous term to the roughness term, and the relative differences of (8)–(10) from the Colebrook root. Churchill’s Darcy form (11) is compared with 64/Re in laminar flow and with Colebrook otherwise. If  $\varepsilon/D \geq 0.05$  the page adds that the roughness height is already a substantial fraction of the diameter. If  $\varepsilon/D \geq 3.7$  it returns the non-existence statement of Section 2.4 and no factor. Editing either input clears the previous result, so a stale root cannot sit beside new numbers.

### 3.2 What was removed

The previous turbulent solver started at  $f = 0.02$  and replaced  $f$  by the square of the right-hand side of (3), stopping on a successive change below  $10^{-7}$  or after thirty iterations. It had no residual. Every  $\text{Re} \geq 2000$  was labeled turbulent. The worked example was described as  $f \approx 0.02$ . Those choices are the ones Section 4 measures. The interactive band on the page still uses the latest reported Darcy factor as a target. It does not enter the residual calculation.

## 4 Benchmarks

### 4.1 The commercial-steel root

Table 1 is the default input. Newton’s method from the Haaland start meets  $|\phi| < 10^{-12}$  in two iterations. The residual at the accepted root is  $-7.11 \times 10^{-15}$ , which is rounding, not a tolerance leftover. The Darcy factor is 0.01856015 and the Fanning factor is 0.00464004. The fully rough asymptote at this roughness is 0.0163109, about 12.1% below the root, and the viscous term is 0.757 times the roughness term. Both terms matter. Calling the result 0.02 would raise (1) by 7.76% with no change in flow rate, diameter, or length.

The old loop is not dramatically wrong on this particular input. It stops at  $f = 0.01856015$  as well, to the digits one would print, but its residual is  $9.83 \times 10^{-8}$  because it stops on a successive difference rather than on  $\phi$ . The damage on the happy path was the prose, not the fifth decimal. The damage off the happy path is the next subsection.

### 4.2 A converged value that is not a root

At  $\text{Re} = 10^5$  and  $\varepsilon/D = 4$  the legacy loop returns  $f = 218.069$  after four iterations, with a successive change below  $10^{-7}$  and a Colebrook residual of 0.135. The right-hand side being squared is negative,

Table 2: Largest relative deviation from the Newton root on 572 turbulent states. The signed value is (approximation – root)/root.

| Formula                             | Maximum  relative error | At Re               | At $\varepsilon/D$   |
|-------------------------------------|-------------------------|---------------------|----------------------|
| Swamee–Jain                         | +2.994%                 | $5 \times 10^3$     | 0.020                |
| Haaland                             | –1.395%                 | $5.946 \times 10^4$ | $4.5 \times 10^{-4}$ |
| Serghides                           | –0.00314%               | $2 \times 10^5$     | 0                    |
| Churchill, as a Colebrook surrogate | +2.990%                 | $5 \times 10^3$     | 0.020                |

which is why a positive update exists at all. Newton’s method returns no root, by the criterion  $\varepsilon/D \geq 3.7$  and by the sign of  $\phi$ . At  $\varepsilon/D = 3.7$  the legacy denominator is zero on the fourth pass. The revised page uses one message for that entire closed end of the roughness axis: there is no positive Colebrook factor, and a roughness several diameters high is not a pipe.

On the 572 turbulent states of the grid below, the Newton residual never exceeds  $9.97 \times 10^{-13}$  in absolute value, and no state takes more than three iterations. The safeguard that halves a non-positive trial step is not what produces those roots. The derivative (6) stays above one, and the Haaland start is already close.

### 4.3 The laminar switch

For the example roughness,  $64/1999 = 0.032016$ , while the Colebrook root at the same Reynolds number is 0.049803, which is 55.6% higher. A solver that flips from (2) to (3) at 2000 changes the reported factor by that amount between two neighboring integers. The jump is not special to this roughness. At  $\varepsilon/D = 0$  the Colebrook root at  $\text{Re} = 1999$  is already 54.5% above  $64/\text{Re}$ , and at  $\varepsilon/D = 0.05$  it is 2.56 times the laminar value. Churchill’s Darcy form at  $\text{Re} = 1999$  and  $\varepsilon/D = 4.5 \times 10^{-4}$  is 0.032059, only 0.133% above  $64/\text{Re}$ . At  $\text{Re} = 2500$  the three numbers separate:  $64/\text{Re} = 0.025600$ , Churchill gives 0.035172, and Colebrook gives 0.046429. Churchill sits between the laminar law and the turbulent implicit law, which is the blend it was written to be [15]. Scoring it there as a 24% Colebrook error would punish it for not making the jump the page no longer makes either.

Above  $\text{Re} = 4000$  the page does make a choice and names it. Colebrook is the reported turbulent factor. The laminar value remains on the result as a comparison, because  $64/(2 \times 10^5) = 0.00032$  is a useful warning that the wrong regime has been selected: it is fifty-eight times smaller than the root in Table 1.

### 4.4 Explicit formulas on one rectangle

The comparison grid is the rectangle  $5 \times 10^3 \leq \text{Re} \leq 10^8$ ,  $0 \leq \varepsilon/D \leq 0.05$ , sampled at eleven roughnesses and at the union of a fixed Reynolds list with 41 logarithmically spaced Reynolds numbers, 572 states in all. Every state has one Newton root. Table 2 records the largest relative deviation of each explicit formula from that root. The location is part of the result. Swamee–Jain and Churchill, judged here as Colebrook approximations, are worst at the rough, low-Reynolds corner  $\text{Re} = 5000$ ,  $\varepsilon/D = 0.02$ . Haaland is worst near the worked-example roughness, at  $\text{Re} = 5.946 \times 10^4$ . Serghides is worst on the smooth law at  $\text{Re} = 2 \times 10^5$ , and “worst” there means 0.00314%.

These are not universal error bounds, and they should not be substituted for the error tables in the source papers or in Brkić’s review [10, 11, 14]. They are the errors of the formulas as implemented on this page, on this rectangle, against a residual-checked root. Within that scope the

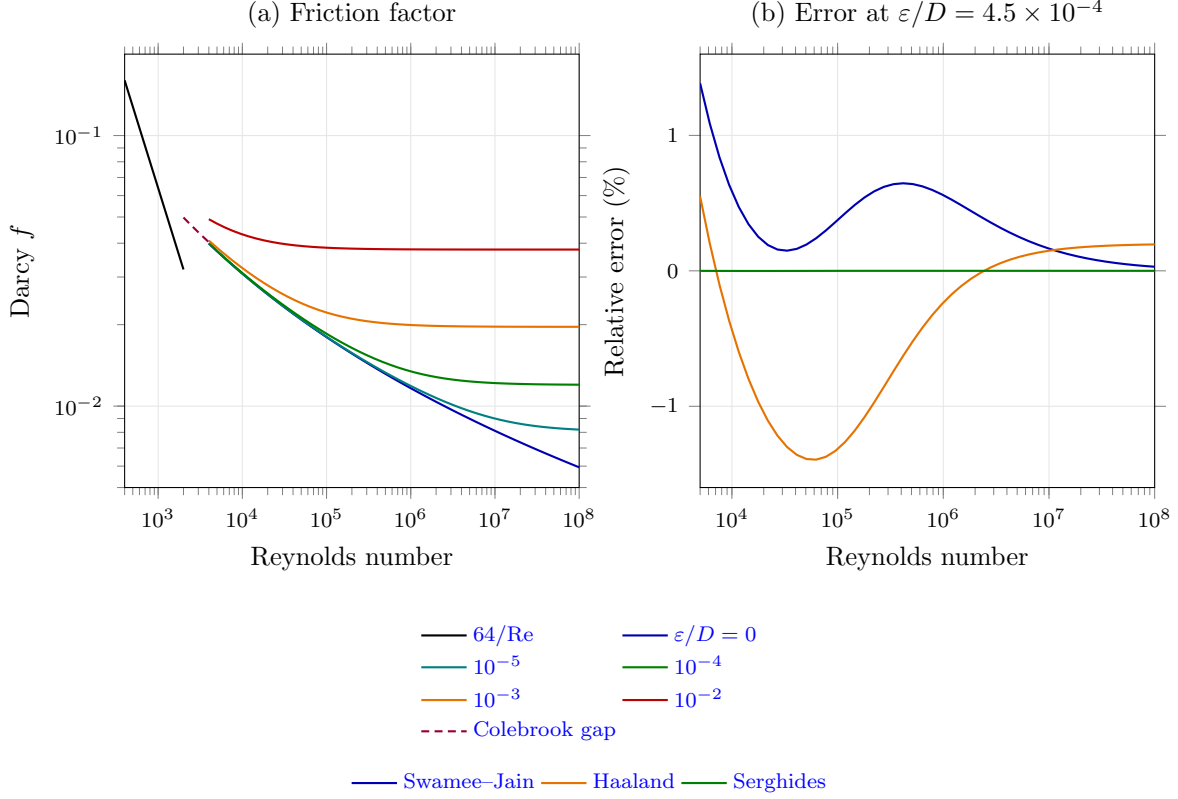


Figure 1: Panel (a): Hagen–Poiseuille factor up to  $Re = 2000$ , and Colebrook–White roots from  $Re = 4000$  to  $10^8$ . The dashed curve is the Colebrook root at  $\epsilon/D = 4.5 \times 10^{-4}$  for  $2000 \leq Re \leq 4000$ , the band in which the calculator reports both laws. Panel (b): relative error of three explicit formulas against the Newton root at the worked-example roughness. Serghides is indistinguishable from the axis on this scale.

practical conclusion is stable. Serghides reproduces Colebrook to a tolerance much tighter than the uncertainty of an equivalent roughness. Swamee–Jain and Haaland are percent-level approximations, the same order as the mistake of replacing 0.01856 by 0.02. They are the right formulas to display next to the root and the wrong formulas to use as a silent replacement for it when the residual is cheap.

Figure 1 draws the same statements. Panel (a) is a Moody-style plot of (2) up to  $Re = 2000$  and of Colebrook roots from  $Re = 4000$  to  $10^8$  at five roughnesses. The dashed segment is the Colebrook root for the example roughness through the band the page refuses to label as turbulent. It does not meet  $64/Re$ . Panel (b) is the relative error of the three turbulent approximations along the example roughness. Serghides lies on the axis at this scale. Swamee–Jain stays positive and Haaland changes sign; neither is a conservative bound.

## 5 Discussion

The numerical claim is narrower than “Newton is the right way to compute friction factors,” and the narrower claim is the one the tables support. On commercial-pipe inputs the old loop and the Newton root agree to engineering decimals, while the residual of the old loop is merely small. The

old loop’s distinctive behavior is elsewhere: it manufactures a positive factor from a non-positive right-hand side, and it describes a 55.6% jump at  $Re = 2000$  as a change from laminar to turbulent physics. A residual test separates those outcomes from a genuine root. So does printing both candidates in the transitional band.

The explicit formulas then fall into place. Once the root costs two or three residual evaluations, the reason to prefer Swamee–Jain or Haaland is convenience of hand calculation, not accuracy. Their percent-level departures on this grid are large enough to matter in the same way that rounding 0.01856 to 0.02 matters, and small enough that they remain useful as a check that the root is in the right neighborhood. Serghides is in a different class. A maximum relative difference of 0.00314% across the rectangle is far inside the uncertainty of any handbook roughness. It is a legitimate explicit stand-in for Colebrook when a residual will not be computed. It is not a reason to omit the residual when the residual is a single logarithm.

Churchill belongs in the result for the regime question, not as a fourth turbulent approximation. At  $Re = 1999$  it tracks Poiseuille. At the worked example it is 0.573% above Colebrook, comparable to Swamee–Jain. Using one formula as if it were the other is how a transitional Reynolds number gets a turbulent label, or a turbulent state gets a laminar one.

None of this tightens the physical uncertainty of  $\varepsilon$ . Equivalent sand-grain roughness is an inferred hydraulic parameter, not a measured average of surface peaks, and Colebrook’s curve was built for commercial pipe rather than for Nikuradse’s sand [2, 3, 7]. A residual of  $10^{-14}$  means the stated equation has been solved. It does not mean the equivalent roughness was the right one, or that the pipe is free of fittings, heat transfer, or an undeveloped entrance. The calculator’s job is to keep that distinction sharp enough that a later pressure-drop calculation can cite a factor, a residual, and a regime without inheriting a false root or a rounded slogan.

## 6 Limitations

The implementation is the round-pipe pair (2) and (3), plus the comparison formulas (8)–(11). It does not solve noncircular ducts, even though a hydraulic diameter is sometimes inserted into the same equations by hand. It does not add minor losses, compressibility, non-Newtonian rheology, or a temperature correction to viscosity. Those effects change  $Re$  or replace  $f$  before this page is asked a question.

The brackets at 2000 and 4000 are conventions. A quiet laboratory pipe can remain laminar well above 2000, and a disturbed industrial inlet can leave laminar flow earlier [4]. Showing both candidates in the band is a disclosure, not a transition model. Churchill’s expression is one published blend across that band; it is not a measurement either.

The grid in Table 2 stops at  $\varepsilon/D = 0.05$  and  $Re = 5 \times 10^3$  on the turbulent side. Outside it, and in particular as  $\varepsilon/D$  approaches 3.7 from below, the factor becomes large and the explicit approximations are not covered by the quoted maxima. The existence statement does cover that end of the axis: there is still one positive root for every  $rr < 3.7$ , and none above it. Relative roughness at or above 0.05 is flagged because the circular-pipe idealization is already strained, not because the algebra has failed.

Serghides, Swamee–Jain, Haaland, and Churchill are transcribed in (8)–(11) and checked against the Newton root. They are not a survey of the dozens of later explicit fits [13, 14]. A fit that is absent here has not been shown to be better or worse.

## 7 Conclusion

A Darcy friction factor is auditable when the equation, the residual, and the regime are the same statement. For  $\text{Re} < 2000$  that statement is  $f = 64/\text{Re}$ . For  $\text{Re} \geq 4000$  it is the unique positive root of Colebrook’s residual  $\phi(g)$ , which Newton’s method finds in at most three iterations on the commercial-pipe rectangle tested here, with  $|\phi| < 10^{-12}$ . For  $2000 \leq \text{Re} < 4000$  the honest statement is that both numbers exist and that the inlet, not the algebra, decides. The root at the default commercial-steel input is  $f = 0.01856015$ , not 0.02. The value  $f = 218$  at a relative roughness of 4 is what a squared right-hand side looks like when no root exists. Reporting  $\phi$  is what makes those two results distinguishable on the page.

## Data and code availability

The calculator is the AgentCalc page [darcy-weisbach-friction-factor-calculator](#), and the released copy is in the [agentcalc.science](#) repository. The benchmark script `generate_darcy_friction_figure.py` and the series plotted in Figure 1 are archived with this note. No laboratory pressure-drop series was analyzed.

## References

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