Browser-Native Lotka–Volterra Simulation: First-Integral Benchmarks for a Predator–Prey Calculator

JJ Ben-Joseph[1,*]
[1]TensorSpace, Inc, Tel Aviv, Israel
[*]jjbj@tensorspace.ai

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Interactive calculator: https://agentcalc.com/lotka-volterra-predator-prey-calculator

Abstract

The classical Lotka–Volterra predator–prey equations are still the usual first model of a population cycle, and they are still routinely integrated in the browser with a single unchecked forward-Euler loop. That combination is a poor one. For positive parameters the continuous system has a first integral, a center equilibrium, and a nested family of periodic orbits; explicit Euler does not stay on those orbits. We document a browser-native calculator that integrates the same vector field with forward Euler, classical fourth-order Runge–Kutta (RK4), and the positivity-preserving symplectic Euler method in logarithmic coordinates, and that returns the first-integral drift together with the equilibrium, the linear period, and the cycle averages. On the benchmark orbit with coexistence equilibrium $(x^*, y^*) = (4, 2.75)$, a fine RK4 reference keeps the prey maximum at 7.272, reproduces the cycle averages to four decimals, and changes the first integral only at roundoff. Forward Euler at a step of 0.1 raises that prey maximum by 60.6% within 40 time units and, carried to time 160, leaves the positive quadrant. The symplectic step at the same size shifts the prey maximum by 0.95% and keeps the invariant error below 7.1×10^{-3} out to time 160. RK4 is the more accurate of the two geometric and classical choices at equal step size. The calculator is useful when the solver, the step, and the invariant are reported with the populations.

1 Introduction

Predator–prey theory began as a pair of independent derivations of the same quadratic system. Lotka obtained undamped oscillations from a general kinetic scheme for interacting species and wrote the two-species case down as a biological example of a conservative oscillation [1, 2]. Volterra, asked to explain Umberto D'Ancona's observation that the proportion of selachians in the Upper Adriatic catch rose while fishing was interrupted during the First World War, derived the same equations and extracted three qualitative laws: the fluctuations are periodic, the averages over a cycle are independent of the initial point on that cycle, and a uniform additional mortality raises the average prey density while it lowers the average predator density [3–6]. The English-language literature has used the model ever since as the starting point for anything more realistic [7–9].

The mathematical object is easy to mistreat numerically, and a browser calculator is a natural place to do so. The coexistence equilibrium of the classical system is a center. Orbits in the positive quadrant are closed curves of a first integral, not spirals into an attractor [10, 11]. Forward Euler, the method most often typed into a teaching script, is weakly unstable on a center: the discrete

orbit spirals outward, amplitudes grow, and a long run can become negative. A plot of that spiral still looks like a predator–prey cycle. Without the first integral on the page, nothing in the interface says that the growth is an artifact.

This note describes the Lotka–Volterra calculator on AgentCalc after that failure mode was treated as a modeling requirement rather than as a disclaimer. The goals are the same ones that make a small simulator citable. The conserved quantity, the period, and the cycle averages have to be visible. The stepper has to be named. And the implementation has to be checked against those analytical facts at a published parameter set, including a step size at which Euler is already misleading. Geometric integration supplies the comparison method. In logarithmic coordinates the system is Hamiltonian, and the symplectic Euler method for that Hamiltonian system stays positive and remains on a nearby closed curve [12]. The benchmarks below separate that geometric property from local accuracy. On this smooth problem RK4, which is not symplectic, is the more accurate integrator at a given step. Symplectic Euler is the method that refuses to spiral.

2 Model

2.1 Vector field

Let $x(t) > 0$ denote prey density and $y(t) > 0$ predator density. The calculator integrates

$$\frac{dx}{dt} = x(\alpha - \beta y), \tag{1}$$

$$\frac{dy}{dt} = y(\delta x - \gamma), \tag{2}$$

with $\alpha, \beta, \gamma, \delta > 0$. The parameter α is the prey per-capita growth rate in the absence of predators, β the predation coefficient, γ the predator per-capita decline in the absence of prey, and δ the conversion of consumed prey into predator growth. The product xy is the mass-action, or Holling type I, encounter rate [13]. Populations and time are whatever units the user intends; the calculator does not nondimensionalize them away, because a reported run should be reproducible from the numbers that were typed.

The axes are invariant. If $x = 0$ then x remains 0 and the predator decays as $e^{-\gamma t}$. If $y = 0$ then y remains 0 and the prey grows as $e^{\alpha t}$. The open positive quadrant is therefore invariant for the differential equation. A numerical state with a negative coordinate is not a population.

2.2 Equilibrium and the linear period

There are two equilibria in the closed quadrant: extinction of both species at the origin, and a coexistence state

$$x^* = \frac{\gamma}{\delta}, \qquad y^* = \frac{\alpha}{\beta}. \tag{3}$$

The Jacobian at coexistence has trace zero and determinant $\alpha\gamma$, so the eigenvalues are the purely imaginary pair $\pm i\sqrt{\alpha\gamma}$. The equilibrium is a linear center. The small-oscillation period

$$T_{\text{lin}} = \frac{2\pi}{\sqrt{\alpha\gamma}} \tag{4}$$

depends on the prey growth rate and the predator death rate only. The interaction coefficients β and δ move the equilibrium, but they do not set the linear frequency. Finite-amplitude periods are

longer than T_{lin}. The calculator prints T_{lin} next to the period measured from successive prey peaks so that lengthening is visible.

The same Jacobian supplies the phase markers of a cycle. From (1), prey density has a turning point only when $y = y^*$, provided $x > 0$. From (2), predator density has a turning point only when $x = x^*$. A numerical prey maximum that occurs far from y^* is evidence that the computed point has left the orbit of the differential equation.

2.3 First integral

Dividing (1) by (2), separating variables, and integrating produces the constant of motion used by Volterra. An equivalent and convenient form is

$$V(x, y) = \delta x - \gamma \ln x + \beta y - \alpha \ln y. \tag{5}$$

Differentiating along a positive solution gives

$$\begin{aligned} \frac{dV}{dt} &= (\delta - \gamma/x)\, x(\alpha - \beta y) + (\beta - \alpha/y)\, y(\delta x - \gamma) \\ &= (\delta x - \gamma)(\alpha - \beta y) - (\alpha - \beta y)(\delta x - \gamma) = 0. \end{aligned}$$

Thus every positive orbit lies on a level curve of V. Those level curves are closed and surround (x^*, y^*). The origin of the logarithm is also why the calculator rejects a zero or negative initial state instead of attempting the integral there.

The change of coordinates $u = \ln x$, $v = \ln y$ turns (1)–(2) into the canonical Hamiltonian system

$$\frac{du}{dt} = -\frac{\partial H}{\partial v}, \qquad \frac{dv}{dt} = \frac{\partial H}{\partial u}, \tag{6}$$

generated by $H(u, v) = \delta e^u - \gamma u + \beta e^v - \alpha v$. In the original variables H and V are the same function. This is the structure a geometric integrator can see [11, 12]. It is also the reason a method that merely stays non-negative is not automatically acceptable: positivity is the orthant, V is the orbit.

2.4 Cycle averages

Volterra's second law follows from the integral. Over any period T of a positive periodic solution,

$$\frac{1}{T}\int_0^T x(t)\, dt = x^*, \qquad \frac{1}{T}\int_0^T y(t)\, dt = y^*. \tag{7}$$

The averages depend on the rates and not on the amplitude. A numerical average close to (7) is meaningful only when it is taken over complete cycles and the orbit has not drifted onto a different level of V. The calculator reports the mean of the accepted steps on the window the user requested, which is the right diagnostic and the wrong place to expect four-decimal agreement unless that window is an integer number of periods.

The third law is the same identity after a change of parameters. Additional per-capita removal rates e and m, the classical idealization of fishing effort, replace (α, γ) by $(\alpha - e, \gamma + m)$. The averaged prey density rises and the averaged predator density falls as effort increases, which is the direction Volterra used for the wartime fishery [3, 6]. The calculator does not add those removal rates. They are a different vector field, with a different V, and they should not be silently present in a tool whose claim is that V can be audited.

3 Browser implementation

3.1 Steppers

The page exposes three one-step methods. Forward Euler is the textbook step,

$$x_{n+1} = x_n + \Delta t\, x_n(\alpha - \beta y_n), \qquad y_{n+1} = y_n + \Delta t\, y_n(\delta x_n - \gamma), \tag{8}$$

and it is the method whose orbit spirals out on this problem [12]. Classical RK4 evaluates the vector field at the standard four stage nodes and combines them with weights $(1, 2, 2, 1)/6$. It is fourth-order on a smooth field [14]. Neither method knows about V.

The third method is the symplectic Euler discretization of the Hamiltonian system (6). One step, written back in (x, y), is

$$x_{n+1} = x_n \exp\big[\Delta t\,(\alpha - \beta y_n)\big], \tag{9}$$

$$y_{n+1} = y_n \exp\big[\Delta t\,(\delta x_{n+1} - \gamma)\big]. \tag{10}$$

The update is explicit, first-order, and symplectic for (6). Because the exponential of a finite number is positive, a positive state remains positive for every finite Δt that does not overflow. The method does not conserve V exactly. Backward error analysis says that it conserves a modified invariant up to an exponentially small defect, so the error in V oscillates at a size set by the step instead of growing in proportion to the length of the run [12]. That is the property the long window in Section 5 is meant to show. It is a different property from having a small local error. At equal Δt, RK4 is much closer to the reference orbit.

3.2 What the page returns

A completed run reports the realized horizon $K\Delta t$, the terminal state, (x^*, y^*), T_{lin}, the prey and predator ranges, the mean spacing of prey maxima, the predator density at the prey maximum, the sample averages, $V(0)$, and

$$\Delta V = V(x_K, y_K) - V(x_0, y_0). \tag{11}$$

The prey-maximum marker is the numerical counterpart of $y(t_{\text{peak}}) = y^*$. The sample averages are the numerical counterpart of (7), with the caveat of Section 2.4. ΔV is the numerical counterpart of (5).

3.3 Safeguards that change the result

Several behaviors of a naive Euler script are rejected because they change the scientific object.

The integrator does not clip a negative coordinate to zero. Clipping replaces (1)–(2) with a different piecewise field and can make a wrecked step look like a population that has bottomed out. If a state ceases to be positive, or ceases to be finite, the run stops and reports the time at which that happened. The symplectic update is the practical way to avoid the positive-orthant failure; reducing Δt is the practical way to avoid it for Euler and RK4.

Inputs must be finite. Both initial populations and all four rates must be positive, so that (x^*, y^*), T_{lin}, and V exist. The step and the horizon must be positive. The step count $K = \text{round}(T/\Delta t)$ has to lie between 1 and 2×10^5. The cap is there because an unbounded loop freezes the page and because a result that never returns is not a small value of ΔV. The realized horizon $K\Delta t$ is what the summary prints, so a horizon that is not an integer multiple of the step is not quietly relabeled.

Editing any field clears the previous summary. A number left on the screen after the rates have been changed is a false result, and this system is sensitive enough that the false result can be a different cycle.

4 Validation design

The benchmark uses the rate set already printed on the calculator,

$$\alpha = 1.1, \quad \beta = 0.4, \quad \gamma = 0.4, \quad \delta = 0.1, \tag{12}$$

which places the coexistence equilibrium at $(x^*, y^*) = (4, 2.75)$ and gives $T_{\text{lin}} = 9.4723$. The reference orbit starts at $(x_0, y_0) = (6, 2)$, off the equilibrium but not near the coordinate axes, and runs to time $T = 40$, a little more than four linear periods. All three browser steppers are applied on that window. The reference trajectory is RK4 at $\Delta t = 5 \times 10^{-4}$. The same steppers, with no clipping, are implemented in the figure script that writes the plotted series, so the table is a test of that algorithm rather than a separate higher-precision code. The reference step is four hundred times finer than the coarsest RK4 step in the table.

Four checks are tied to the claims of the page.

1. **Invariant and amplitude.** ΔV, the prey range, and the peak-to-peak period are compared across methods and step sizes on $[0, 40]$.

2. **Phase condition and linear period.** On the reference orbit, the predator density at each prey maximum is compared with y^*. A second orbit, started at $(4.2, 2.75)$, checks T_{lin} in the small-amplitude regime.

3. **Cycle averages.** Trapezoidal means of the reference solution are taken from the first prey maximum to the fifth, which is four complete periods, and compared with (7).

4. **Secular drift.** Euler and the symplectic method are continued at $\Delta t = 0.1$ to times 80 and 160, to distinguish an invariant error that grows with the horizon from one that stays bounded while the amplitude stays put.

5 Results

5.1 Reference orbit

The fine RK4 run finishes at time 40 with prey between 1.897 and 7.272 and predators between 1.791 and 4.002. Its peak-to-peak period is 9.711, which is 2.52% longer than T_{lin}, as expected for a finite-amplitude cycle of (5). The terminal invariant error is -5.3×10^{-15}.

The phase condition is sharp. The first three prey maxima occur at times 1.135, 10.845, and 20.556, and the predator densities there are 2.7502, 2.7499, and 2.7501 against $y^* = 2.75$. Started instead at $(4.2, 2.75)$, the same stepper has period 9.4735, a relative difference of 0.013% from $T_{\text{lin}} = 9.4723$. The linearization (4) is the right small-amplitude benchmark, and the calculator's displayed T_{lin} is not a stand-in for the period of a large cycle.

Over the four complete cycles from the first prey peak to the fifth, a span of 38.843 time units, the trapezoidal means are 4.0000 and 2.7500. That is (7) to the digits reported. The sample mean over the slightly longer window $[0, 40]$ is not this number, because $40/9.711$ is not an integer. The distinction is worth making on the page: the law is about cycles, not about whatever horizon was convenient.

At the equilibrium itself the symplectic step with $\Delta t = 0.05$ remains at $(4, 2.75)$ with $\Delta V = 0$ through time 10. The vector field vanishes there, and the exponential update is idle. Departure from equilibrium in a user run is not the equilibrium formula being wrong.

Table 1: Solver benchmark for $\alpha = 1.1$, $\beta = 0.4$, $\gamma = 0.4$, $\delta = 0.1$, $(x_0, y_0) = (6, 2)$, and horizon $T = 40$. The prey-maximum error is relative to the RK4 reference at $\Delta t = 5 \times 10^{-4}$. Every run in this table remained positive.

Solver	Δt	Max prey	Min prey	Period	ΔV	Max-prey error (%)
Euler	0.20	25.995	0.001	11.133	2.84	+257
Euler	0.10	11.682	0.582	10.067	0.383	+60.6
Euler	0.05	8.936	1.240	9.850	0.119	+22.9
Euler	0.01	7.624	1.772	9.733	0.0169	+4.84
RK4	0.20	7.271	1.897	9.667	-1.56×10^{-6}	−0.01
RK4	0.05	7.272	1.897	9.700	-2.80×10^{-9}	0.00
Symplectic Euler	0.20	7.415	1.841	9.733	6.46×10^{-3}	+1.97
Symplectic Euler	0.10	7.341	1.869	9.700	3.14×10^{-3}	+0.95
Symplectic Euler	0.05	7.306	1.883	9.717	1.56×10^{-3}	+0.47
RK4 reference	0.0005	7.272	1.897	9.711	-5.3×10^{-15}	reference

5.2 Step size, amplitude, and invariant drift

Table 1 compares the three methods on $[0, 40]$. Relative prey-maximum error is measured against the reference maximum 7.272.

Euler is monotonically approaching the reference as Δt decreases, and it is not close at the steps a user is likely to try first. At $\Delta t = 0.1$ the prey peak is 60.6% high, the trough has fallen from 1.897 to 0.582, and the period has stretched from 9.711 to 10.067. At $\Delta t = 0.2$ the prey maximum is 26.0 and the minimum is 1.0×10^{-3}. The orbit is still positive on this particular window, so a script that only checks for negative numbers would accept it. $\Delta V = 2.84$ would not.

RK4 at $\Delta t = 0.2$, a step large enough to ruin Euler, keeps the prey maximum within 0.01% of the reference and holds $|\Delta V|$ at 1.6×10^{-6}. At the calculator's default $\Delta t = 0.05$ the invariant error is 2.8×10^{-9}. For this non-stiff field, order is doing the work that a smaller step would otherwise have to do.

Symplectic Euler is less accurate than RK4 at the same step and much more faithful than forward Euler. At $\Delta t = 0.1$ the prey maximum is high by 0.95% rather than by 60.6%, and $\Delta V = 3.1 \times 10^{-3}$. The error shrinks as the step shrinks, but only about linearly, which is the order of the method. Figure 1 shows the geometric content of the same comparison: the reference and the symplectic step at $\Delta t = 0.1$ trace a closed curve around $(4, 2.75)$, while Euler at that step has wound outward by time 40.

5.3 The error that grows

Accuracy on a short window is not the same statement as the character of the error. Continued at $\Delta t = 0.1$, forward Euler has $\Delta V = 0.383$ at time 40, $\Delta V = 1.92$ at time 80, and a prey maximum that has grown from 11.68 to 24.80. By time 160 the Euler trajectory has left the positive quadrant, after recording a prey value of 119.7. The identical step of the symplectic method keeps the prey maximum at 7.341 from time 40 through time 160, and the largest excursion of V from its initial value on that whole interval is 7.01×10^{-3}. The terminal value at time 160 is 6.6×10^{-4}, smaller than the excursion, which is the signature of an oscillating invariant error rather than of a drift proportional to T.

RK4 at $\Delta t = 0.2$ has not joined that spiral by time 160: the prey range remains $[1.897, 7.272]$ and $\Delta V = -5.8 \times 10^{-6}$. The drift is no longer zero, and it has grown relative to the value 1.6×10^{-6} at time 40, so a non-symplectic method of high order is not a certificate for an arbitrarily long run.

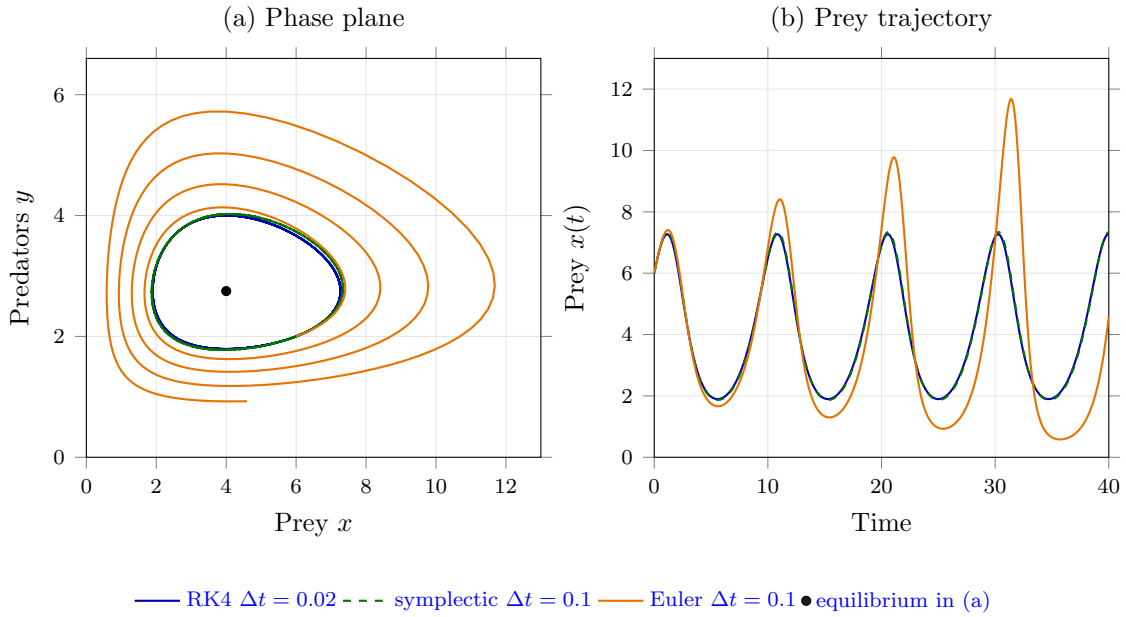


Figure 1: Benchmark orbit for $\alpha = 1.1$, $\beta = 0.4$, $\gamma = 0.4$, $\delta = 0.1$, and $(x_0, y_0) = (6, 2)$. The reference curve is RK4 at $\Delta t = 0.02$, which on this window is indistinguishable at plotting scale from the $\Delta t = 5 \times 10^{-4}$ reference of Table 1. The symplectic Euler orbit stays near that curve. Forward Euler spirals outward, and the prey peaks in panel (b) grow.

On every window in this study it is still the most accurate of the three, by orders of magnitude, at the steps where Euler has already changed the ecology of the output.

6 Discussion

The numerical lesson is narrower than "use a symplectic method," and the narrower claim is the one the table supports. Explicit Euler is the wrong default for this equation because the equilibrium is a center. A step of 0.1 on a period of about 9.7 looks modest, about one percent of a cycle, and it still inflates the prey peak by more than half within four cycles. RK4 removes that error for the horizons and steps a browser session actually uses. Symplectic Euler is the method that keeps the qualitative theorem true when the step is coarse or the run is long: the orbit remains a closed curve in the positive quadrant, and ΔV does not march off with T. A calculator that offers only Euler, or that offers RK4 without printing ΔV, cannot show a user which of those statements they are relying on.

The ecological lesson is older than the numerical one, and it limits what any of these runs can mean. The center is not hyperbolic. A small change in the vector field, such as a logistic term in the prey equation or a saturating functional response, destroys the nested family of periodic orbits and replaces them with a spiral point or a stable limit cycle [13, 15–17]. Gause's competition and predation experiments were already an empirical argument against treating the neutral cycle as a robust description of a closed culture [18]. The Canada lynx series, the other standard picture attached to this model, is a documented ten-year oscillation in fur returns [19], but the mechanism that sustains it involves the hare's food plants as well as predation [20, 21]. A closed level curve of V is not a forecast of that series, and fitting $(\alpha, \beta, \gamma, \delta)$ to it would spend the model's structural freedom on a cycle the model cannot stabilize.

What the calculator can support is a precise forward scenario: these rates, this initial state, this stepper, this step, this horizon, and this ΔV. That is enough for a methods paragraph, a classroom comparison of amplitude against T_{lin}, or a check that a coarser code has not wandered off the integral. It is not an inference about a real community, and the page should not be asked to become one by adding a regression to a fur-return table.

7 Limitations

The implementation is the quadratic mass-action system and nothing else. There is no carrying capacity, no Holling type II or type III response, no predator interference, no explicit harvest, no seasonal forcing, no age structure, and no noise. Each of those extensions changes the first integral or removes it [8–10]. Putting them behind the same button would make ΔV refer to an equation the user was no longer integrating.

The symplectic stepper is first-order. Table 1 is the quantitative version of that sentence: at $\Delta t = 0.2$ it is already two percent high on the prey maximum, which is a real bias even though the bias does not grow into a spiral. Overflow of the exponential at an extreme step is reported as a non-finite state rather than repaired. The sample averages are not cycle averages unless the user chooses a window that closes. The tool does not estimate parameters, does not return a confidence interval, and does not accept a time series.

8 Conclusion

A Lotka–Volterra calculator can be checked, because the differential equation provides the check. The coexistence equilibrium, the period $2\pi/\sqrt{\alpha\gamma}$, the phase condition at the prey peak, the cycle averages, and the first integral V are all available before any time step is chosen. On a published benchmark orbit, fine RK4 meets those identities at roundoff, RK4 at a coarse step still meets them for practical purposes, symplectic Euler keeps a nearby periodic orbit with a bounded invariant error, and forward Euler at $\Delta t = 0.1$ produces a different and growing cycle that eventually leaves the positive quadrant. Reporting the stepper and ΔV is what makes those four outcomes distinguishable on the page.

Data and code availability

The calculator is the AgentCalc page lotka-volterra-predator-prey-calculator, and the released copy is in the agentcalc_science repository. The benchmark script `generate_lotka_volterra_figure.py` and the series plotted in Figure 1 are archived with this note. No observational population series was analyzed.

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