Browser-Native Stokes Settling: A Signed Reynolds Number and a Drag-Corrected Terminal Speed

JJ Ben-Joseph[1,*]
[1]TensorSpace, Inc, Tel Aviv, Israel
[*]jjbj@tensorspace.ai

September 22, 2026

Interactive calculator: https://agentcalc.com/stokes-settling-velocity-calculator

Abstract

The terminal speed of a small rigid sphere is Stokes's balance of viscous drag against effective weight. The same balance at a higher particle Reynolds number is an implicit equation for the drag coefficient. A browser calculator that formed the Reynolds number from the signed Stokes velocity treated every rising sphere as creeping flow, because a negative number is less than 0.1. For a rigid sphere of density $800\,\mathrm{kg\,m^{-3}}$ and radius 1 mm in water, that signed Reynolds number is -872 and the Stokes speed is $-0.436\,\mathrm{m\,s^{-1}}$. The Schiller–Naumann root is a Reynolds number of 152.09 and a rise of $0.07604\,\mathrm{m\,s^{-1}}$. Stokes's formula is 5.734 times too fast, and the old label still said the Stokes regime was valid. The replacement uses the speed in the Reynolds number. Below a Stokes Reynolds number of 0.1 the reported terminal velocity is the Stokes velocity. Otherwise it is the unique positive root of the Schiller–Naumann residual, or a Newtonian coefficient 0.44 once that root would reach 1000. On the default mineral sphere, radius $10\,\mu$m and density $2500\,\mathrm{kg\,m^{-3}}$ in water, the two speeds differ by 0.472% and the residual is zero after two Newton iterations. At $100\,\mu$m the Stokes speed is 42.69% high. The derivative of the residual is positive, so the root is unique, and the iteration starts from the Stokes Reynolds number itself.

1 Introduction

Stokes's formula is the first quantitative answer to a practical question: how fast does a small sphere sink, or rise, once drag has caught up with its buoyant weight? The drag he derived for slow motion is linear in speed, the effective weight is fixed, and the terminal velocity follows at once [1]. The formula is short enough to type into a web form, and it is wrong, quietly, as soon as the inertia of the fluid around the particle is no longer negligible. The usual warning is a Reynolds number. The usual threshold quoted for the linear law is a particle Reynolds number of about 0.1.

A threshold only works if the number plugged into it is the Reynolds number. The previous AgentCalc page computed

$$\mathrm{Re}_{\mathrm{signed}} = \frac{2\rho_f v_S r}{\eta}$$

from the signed Stokes velocity v_S and labeled the motion "Laminar (Stokes valid)" whenever that quantity was below 0.1. A sphere heavier than the fluid has $v_S > 0$, so the test can fail honestly. A sphere lighter than the fluid has $v_S < 0$. Every negative number is below 0.1, so every rising sphere

was labeled valid, including one whose speed-based Reynolds number is hundreds. The label was not a rounding of a borderline case. It was the sign of buoyancy.

The second gap was what the page did after the label. It always reported the Stokes speed. A Reynolds number of 6.5, for a 100 μm mineral sphere in water, was enough to mark the regime as outside Stokes flow and not enough to change the velocity the user was given. The correction at that point is not a refinement of the last digit. The Schiller–Naumann terminal speed is 42.69% slower.

This note documents the Stokes settling calculator after both defects were removed. The Reynolds number is formed from the speed. The reported terminal velocity is the Stokes velocity only while that Reynolds number is below 0.1. Otherwise it is a residual-checked root of a standard rigid-sphere drag law, and the Stokes velocity is kept beside it as a comparison. Direction is a separate word, settles or rises, and it is not folded into the regime test. The benchmarks below are three spheres on which those statements can be checked by hand, and a radius sweep on which the departure from the radius-squared rule is visible.

2 Model

2.1 Stokes balance

For a rigid sphere of radius r and density ρ_p in an unbounded Newtonian fluid of density ρ_f and dynamic viscosity η, the buoyant weight is the weight minus the weight of the displaced fluid,

$$F_g = \frac{4}{3}\pi r^3(\rho_p - \rho_f)g. \tag{1}$$

Stokes's drag for steady translation at speed v in the creeping-flow limit is $6\pi\eta r v$, opposing the motion [1]. Setting the magnitudes equal and restoring the sign of the density difference gives the terminal velocity

$$v_{\mathrm{S}} = \frac{2}{9}\frac{(\rho_p - \rho_f)gr^2}{\eta}. \tag{2}$$

The radius is squared. Doubling it multiplies the Stokes speed by four, and multiplying the viscosity by ten divides that speed by ten. Both statements are properties of (2). They are properties of the terminal speed only while the linear drag law remains the right one.

Equation (2) assumes a rigid surface, a constant viscosity, a particle that has already reached terminal speed, and a fluid that extends far from the particle. A gas bubble or a liquid drop has an internal viscosity and a mobile surface. Its low-Reynolds drag is not $6\pi\eta r v$, and Clift, Grace, and Weber treat drops and bubbles apart from rigid spheres [2]. The rising sphere in the benchmark below is rigid. It is not an oil drop and not a bubble.

2.2 Reynolds number

Reynolds distinguished the direct and sinuous motion of water by a dimensionless ratio of inertia to viscosity [3]. For a sphere the same ratio is formed with the diameter $2r$ and the speed relative to the fluid,

$$\mathrm{Re} = \frac{2\rho_f\,|v|\,r}{\eta}. \tag{3}$$

The absolute value is the whole of the correction to the old test. Equation (3) is positive for a rising sphere. The sign of v still matters, because it says whether the particle settles or rises, but it is not part of the regime test. The calculator reports Stokes's velocity as the terminal speed when the Reynolds number of that velocity satisfies $\mathrm{Re}_{\mathrm{S}} < 0.1$. The threshold is the conventional edge of the linear law, not a theorem that the drag suddenly changes at one thousandth of a unit.

2.3 A drag coefficient and one algebraic group

Once inertia contributes to the drag, the force is written

$$F_D = \frac{1}{2} C_D \, \rho_f v^2 \, (\pi r^2), \tag{4}$$

with a dimensionless coefficient C_D that depends on Reynolds number for a smooth rigid sphere [2]. Balancing (4) against the magnitude of (1) and eliminating v with (3) produces a closed equation for the unknown Reynolds number. The algebra is short and worth keeping visible, because every later numerical claim is a root of this equation. From (3), $|v| = \mathrm{Re}\,\eta/(2\rho_f r)$. Substitute that speed into the balance $C_D \rho_f v^2/2 = (4/3) r |\rho_p - \rho_f| g$. The powers of v and ρ_f collapse to

$$C_D \, \mathrm{Re}^2 = K, \qquad K = \frac{32}{3} \frac{\rho_f \, r^3 \, |\rho_p - \rho_f| \, g}{\eta^2}. \tag{5}$$

The group K is known from the inputs. It does not depend on the unknown speed. Stokes's law is the special case $C_D = 24/\mathrm{Re}$, which turns (5) into $\mathrm{Re} = K/24$. That Reynolds number is exactly Re_S, the one computed from (2) and (3). The identity is a useful check: if a program's Stokes Reynolds number and its $K/24$ disagree, one of the two formulas has been transcribed wrongly.

2.4 Schiller–Naumann

For a rigid sphere at Reynolds numbers above the Stokes regime and below about 1000, a standard empirical fit is the Schiller–Naumann correlation [2, 4, 5]

$$C_D = \frac{24}{\mathrm{Re}} \left(1 + 0.15 \, \mathrm{Re}^{0.687}\right). \tag{6}$$

Cheng compared a family of explicit settling formulas and drag fits, including this one [5]. This calculator does not adopt an explicit settling formula from that comparison. It solves (6) inside (5). The percentages later in the note are residuals and relative differences for this page. They are not a reproduction of Cheng's tables.

Inserting (6) into (5) gives the scalar residual

$$\phi(\mathrm{Re}) = 24 \, \mathrm{Re} \left(1 + 0.15 \, \mathrm{Re}^{0.687}\right) - K. \tag{7}$$

The derivative is

$$\phi'(\mathrm{Re}) = 24 \left(1 + 0.15 \times 1.687 \, \mathrm{Re}^{0.687}\right), \tag{8}$$

which is greater than 24 for every $\mathrm{Re} > 0$. A strictly increasing continuous function crosses zero at most once. Here $\phi(0^+) = -K$ and $\phi(\mathrm{Re}) \to \infty$ as $\mathrm{Re} \to \infty$, so if $K > 0$ there is exactly one positive root. Newton's method is the natural iteration. It starts at $\mathrm{Re} = K/24$, the Stokes Reynolds number, where ϕ is already positive because the extra term $0.15 \, \mathrm{Re}^{0.687}$ makes the Schiller–Naumann drag larger than Stokes drag. The first step therefore moves downward, toward a slower particle. The implementation stops when $|\phi| < 10^{-12} \max(1, K)$, halves a step that would leave the positive axis, and gives up after 30 iterations. On the cases in this note it does not get past six.

The terminal speed is then recovered from the root by (3), with the sign of $\rho_p - \rho_f$:

$$v = \mathrm{sign}(\rho_p - \rho_f) \, \frac{\mathrm{Re}\,\eta}{2\rho_f r}. \tag{9}$$

If the Schiller–Naumann root is at $\mathrm{Re} \geq 1000$, that fit is being used outside the range in which it is usually quoted. The page then replaces it with the constant Newtonian coefficient $C_D = 0.44$, so

$\mathrm{Re} = \sqrt{K/0.44}$, and it says that it has done so. At $\mathrm{Re} = 1000$ the Schiller–Naumann coefficient is 0.438288, against 0.44. For the mineral-in-water pair used below, the Schiller–Naumann root reaches 1000 at a radius of $1.408 \times 10^{-3}\,\mathrm{m}$. Switching to 0.44 at that point lowers the speed by 0.195%. The kink is a property of the piecewise definition, and it is smaller than the modeling error of either formula. The constant 0.44 is a coarse description of the broad range in which sphere drag varies slowly. It is not a model of the later drag crisis, in which the coefficient of a smooth sphere falls sharply [2].

2.5 Oseen's correction, for comparison

The first inertial correction to Stokes drag is Oseen's, $C_D = (24/\mathrm{Re})(1+3\mathrm{Re}/16)$, in the normalization surveyed by Clift, Grace, and Weber [2]. Substituted into (5) it becomes the quadratic $4.5\,\mathrm{Re}^2 + 24\,\mathrm{Re} - K = 0$, with positive root

$$\mathrm{Re} = \frac{-24 + \sqrt{576 + 18K}}{9}. \tag{10}$$

Equation (10) is a low-Reynolds expansion, not the law the calculator reports. It is computed in the benchmark so the Schiller–Naumann root can be seen departing from it once the Reynolds number is no longer small. At the $10\,\mu$m sphere the Oseen Reynolds number is 0.0065320, between the Stokes value 0.00654 and the Schiller–Naumann value 0.0065093. At the $100\,\mu$m sphere the Oseen root is 3.813, the Schiller–Naumann root is 4.583, and the Stokes value is 6.54. The expansion has already left the empirical fit, which is why it is not the reported speed.

3 What the page returns

The form takes radius, the two densities, viscosity, and gravitational acceleration, all in SI units. The default is the worked example of the next section: $r = 10^{-5}\,\mathrm{m}$, $\rho_p = 2500\,\mathrm{kg\,m^{-3}}$, $\rho_f = 1000\,\mathrm{kg\,m^{-3}}$, $\eta = 10^{-3}\,\mathrm{Pa\,s}$, and $g = 9.81\,\mathrm{m\,s^{-2}}$. The value 9.81 is the page default, not a standard-gravity constant of 9.80665. Changing it would move the reference speeds below. Empty or non-positive inputs produce no velocity. Equal densities produce a neutrally buoyant result, with both speeds and both Reynolds numbers equal to zero and no drag coefficient.

For a density difference, the result contains the direction, the Stokes velocity and its Reynolds number, the drag-corrected velocity and its Reynolds number, the drag coefficient, and either the Schiller–Naumann residual and iteration count or an explicit statement that $C_D = 0.44$ was used because the Schiller–Naumann root had reached 1000. The reported terminal velocity follows one rule. If $\mathrm{Re_S} < 0.1$ and the drag law is still Schiller–Naumann, the reported speed is v_S. Otherwise the reported speed is the drag-corrected speed. Both numbers are always shown, so the rule can be audited against the arithmetic.

The old page could not make that distinction for a rising particle. Its regime test was $\mathrm{Re_{signed}} < 0.1$. The new page never forms that signed ratio for the label. A clarifier mini-game on the same HTML page is a separate animation with its own Stokes timing. It is not the terminal speed in Table 1, and it is not part of the released science-page copy of the calculator.

4 Three spheres

Table 1 is the check. The fluid is water, $\rho_f = 1000\,\mathrm{kg\,m^{-3}}$ and $\eta = 10^{-3}\,\mathrm{Pa\,s}$, and $g = 9.81\,\mathrm{m\,s^{-2}}$. The first two spheres are mineral grains, $\rho_p = 2500\,\mathrm{kg\,m^{-3}}$. The third is rigid and lighter than water, $\rho_p = 800\,\mathrm{kg\,m^{-3}}$.

Table 1: Stokes and drag-corrected terminal states for three rigid spheres in water at $g = 9.81\,\mathrm{m\,s^{-2}}$. The relative difference is $(v_S - v)/v$. The legacy column is the old test $\mathrm{Re}_{\mathrm{signed}} < 0.1$.

	$10\,\mu$m mineral	$100\,\mu$m mineral	1 mm, $\rho_p = 800$
Stokes velocity ($\mathrm{m\,s^{-1}}$)	3.270×10^{-4}	0.03270	-0.436
Re_S	0.00654	6.54	872
Corrected velocity ($\mathrm{m\,s^{-1}}$)	3.2546×10^{-4}	0.022917	-0.076043
Corrected Re	0.0065093	4.5834	152.087
C_D	3704.45	7.4717	0.90478
Newton iterations	2	4	6
Residual ϕ	0	0	-3.638×10^{-12}
$(v_S - v)/v$	+0.472%	+42.69%	+473.4%
Legacy label "Stokes valid"	yes	no	yes

The $10\,\mu$m row is exact in the Stokes column. With $r = 10^{-5}\,\mathrm{m}$ and a density difference of $1500\,\mathrm{kg\,m^{-3}}$,

$$v_S = \frac{2}{9} \cdot 1500 \cdot 9.81 \cdot 10^{-10}/10^{-3} = 3.270 \times 10^{-4}\,\mathrm{m\,s^{-1}},$$

and (3) then gives $\mathrm{Re}_S = 0.00654$ with no further rounding. The group in (5) is $K = 0.15696$, and $K/24 = 0.00654$. Schiller–Naumann's equation moves the Reynolds number to 0.00650927 and the speed to $3.25464 \times 10^{-4}\,\mathrm{m\,s^{-1}}$. The relative difference is 0.472%. Two Newton iterations leave a residual of zero at double precision, against a tolerance of 10^{-12}. Because $\mathrm{Re}_S < 0.1$, the reported terminal velocity on the page is the Stokes value, and the drag-corrected value is the comparison. The old label agrees with that choice. This is the sphere the form opens on.

The $100\,\mu$m row is the same particle with the radius multiplied by ten, so the Stokes speed is multiplied by one hundred: $0.03270\,\mathrm{m\,s^{-1}}$ and $\mathrm{Re}_S = 6.54$. The Schiller–Naumann root is $\mathrm{Re} = 4.58337$, $v = 0.0229168\,\mathrm{m\,s^{-1}}$, and $C_D = 7.4717$, after four iterations with a zero residual. Stokes's formula is 42.69% fast. The reported speed is the drag-corrected one. The legacy test also rejects Stokes flow here, because the Stokes velocity is positive. The failure mode of the old page was not this row. It was the next one.

The rising sphere has radius 1 mm and density $800\,\mathrm{kg\,m^{-3}}$. The same arithmetic gives $v_S = -0.436\,\mathrm{m\,s^{-1}}$ exactly at $g = 9.81$, and the signed ratio used by the old page is -872. That number is less than 0.1, so the old label was "Laminar (Stokes valid)." The speed-based Stokes Reynolds number is 872. The Schiller–Naumann root is $\mathrm{Re} = 152.087$, $C_D = 0.90478$, and $v = -0.0760434\,\mathrm{m\,s^{-1}}$, after six iterations. The residual is -3.638×10^{-12}, below the tolerance $10^{-12} \times K$ with $K = 20928$. The Stokes rise is faster than the drag-corrected rise by a factor of 5.734, which is a relative difference of +473.4% on the sign convention of the table. Nothing about the negative sign made the linear drag law more accurate. It only made the comparison $\mathrm{Re}_{\mathrm{signed}} < 0.1$ true.

A fourth state, not in the table, checks the piecewise switch. A mineral sphere of radius 1.5 mm in the same water has a Schiller–Naumann root above 1000, so the reported law is $C_D = 0.44$. The corrected Reynolds number is 1097.25 and the Stokes speed is 20.12 times the corrected speed. At a radius of 2 mm the Stokes speed is $13.08\,\mathrm{m\,s^{-1}}$, the corrected speed is $0.4223\,\mathrm{m\,s^{-1}}$, and the ratio is 30.97. Those speeds are far outside creeping flow. They are still roots of (5), not measurements.

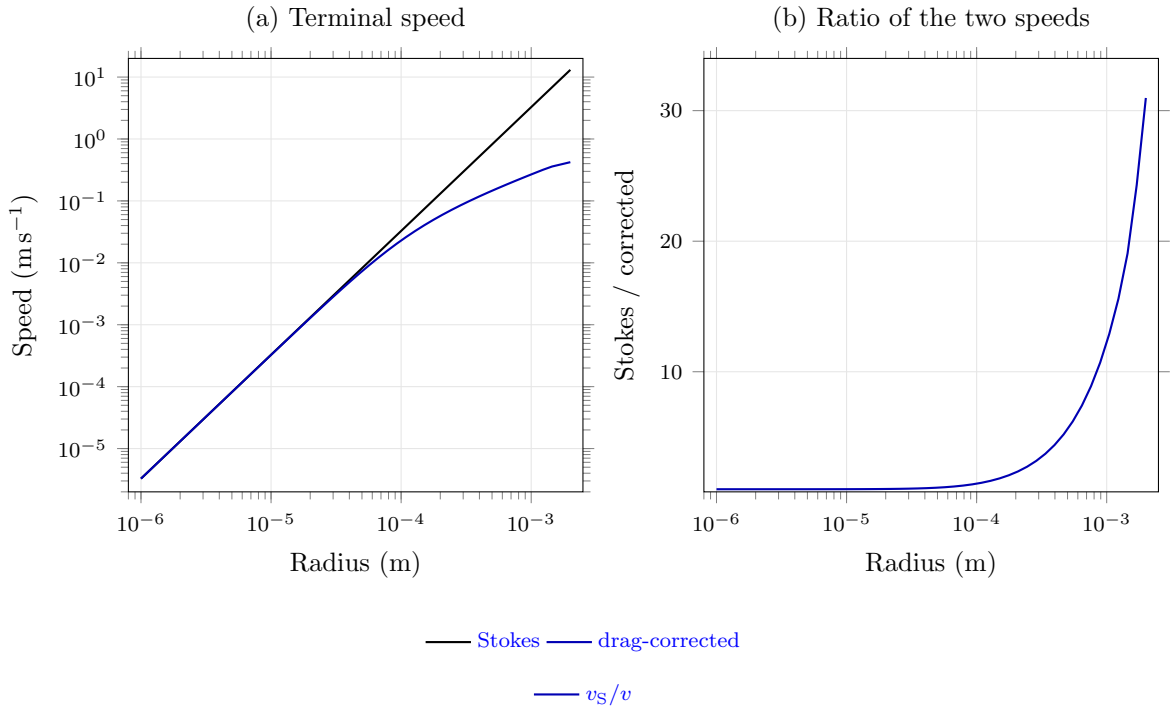

Figure 1: A mineral sphere, $\rho_p = 2500\,\mathrm{kg\,m^{-3}}$, in water at $\eta = 10^{-3}\,\mathrm{Pa\,s}$ and $g = 9.81\,\mathrm{m\,s^{-2}}$. Panel (a): absolute Stokes speed and absolute drag-corrected terminal speed. Panel (b): their ratio. The drag-corrected branch uses Schiller–Naumann's equation until the root reaches $\mathrm{Re} = 1000$, at a radius of $1.408 \times 10^{-3}\,\mathrm{m}$, and $C_D = 0.44$ after that. The one-percent departure of the ratio from 1 is at $1.442 \times 10^{-5}\,\mathrm{m}$.

5 Departure along a radius sweep

Figure 1 draws the mineral-in-water pair from a radius of 1 μm to 2 mm, forty-eight radii spaced evenly in the logarithm. Panel (a) shows $|v_S|$ and the absolute drag-corrected speed. Panel (b) shows the ratio v_S/v, which is positive because the two speeds share a sign.

At 1 μm the ratio is 1.000041. Stokes's formula and Schiller–Naumann's equation are the same number for any purpose this calculator has. The ratio reaches 1.01, a one-percent departure, at a radius of $1.442 \times 10^{-5}\,\mathrm{m}$, where $\mathrm{Re}_S = 0.01961$. That is still below the reporting threshold of 0.1, so the page continues to report the Stokes speed and continues to show the one-percent-level comparison beside it. By 100 μm the curves in panel (a) have separated and the ratio in panel (b) is 1.427. From there to 2 mm the Stokes curve keeps the r^2 slope on the log-log axes. The corrected curve bends as C_D stops falling like 24/Re and levels toward a constant. The piecewise change to $C_D = 0.44$ at 1.408 mm shifts the corrected speed by 0.195% and does not show at the scale of the figure.

The figure is also the reason the radius-squared rule is qualified on the page. Inside the Stokes regime, doubling the radius quadruples the speed. Along panel (a), doubling the radius from 100 μm to 200 μm does not quadruple the drag-corrected speed, because the drag law itself depends on the speed. The r^2 statement remains true of (2), and the plot shows when (2) has stopped being the terminal velocity.

6 Discussion

The numerical claim is narrower than "Schiller–Naumann is the correct drag law," and the narrower claim is the one Table 1 supports. The old page and the new page agree, to the digits a user would copy, on a 10 μm mineral sphere in water. They disagree on what a Reynolds number of -872 means. A regime test that uses a signed velocity will accept every rising sphere, and the acceptance is independent of how large the speed is. Replacing the signed velocity by its magnitude makes the 1 mm sphere fail the Stokes test for the same reason the 100 μm settling sphere fails it.

Once the test can fail, the page has to return a different speed or the test is only a warning under an unchanged formula. Equation (7) is a convenient different speed because it has one positive root, an initial guess that is already on the page, and a residual the result can show. Two, four, and six Newton iterations, with residuals at or below 10^{-11} on the three headline spheres, are enough to separate a solved drag balance from a Stokes formula that was merely labeled. The residual does not make (6) true of a laboratory sphere. It makes the reported number a root of the equation the page names.

The Oseen root is the right companion at $\mathrm{Re}_S = 0.00654$ and the wrong companion at $\mathrm{Re}_S = 6.54$. Keeping it out of the reported result, and in the benchmark, is the same kind of distinction as keeping v_S on screen after the page has stopped reporting it. A user who wants the linear law can still read it. A user who wants a low-Reynolds expansion can compute (10) from K. The number in the headline is the one the regime rule selected.

The piecewise constant 0.44 is the least elegant part of the rule, and the 0.195% jump at 1.408 mm is the measure of that inelegance for this fluid pair. Smoothing it would hide a change of formula. Leaving it, and saying when $C_D = 0.44$ is in use, keeps the reported law identifiable. A calculator that silently blended the two coefficients would have a prettier curve in Figure 1 and a harder sentence in the result.

7 Limitations

The implementation is a single rigid sphere in an unbounded Newtonian fluid. Several effects that dominate real suspensions are outside it.

A nearby wall slows a settling sphere. The correction depends on the ratio of the radius to the vessel radius and is tabulated for several geometries by Clift, Grace, and Weber [2]. The page assumes that ratio is negligible. A particle timed in a narrow cylinder will be slower than either column of Table 1.

Particles in a crowd hinder one another. Richardson and Zaki expressed the settling speed of a uniform suspension as the single-particle speed multiplied by a power of the voidage [6]. The exponent depends on Reynolds number. This page is the single-particle factor in that product. For a concentrated slurry it is larger than the bulk settling speed.

Shape works in the same direction for most grains. A flake, a rod, or an angular fragment has more drag than a sphere of the same volume, so (2) and (6) are both fast relative to the irregular particle. The page has no shape factor.

Fluid particles are a different drag law even at low Reynolds number, because the internal fluid circulates. The rising benchmark must not be read as the rise speed of a bubble or of an oil drop of density 800 kg m^{-3} [2].

Very small particles have a further competition that terminal velocity does not describe. As the radius falls well below a micrometre in water, Brownian displacement over the time of an observation becomes comparable to the distance the particle would settle. The 1 μm end of Figure 1 is still a

settling calculation. It is not a warrant that diffusion can be ignored for a much finer colloid.

The threshold 0.1, the exponent 0.687, the coefficient 0.15, and the constant 0.44 are conventions of a fitted description, not derived constants [2, 4, 5]. Another rigid-sphere fit would move the corrected column of Table 1 by more than the Newton residual. The residual certifies the root of this residual. Agreement with a measured settling speed is a separate experiment, and this note does not report one. Gravitational acceleration is an input. The reference numbers use $9.81\,\mathrm{m\,s^{-2}}$ because that is the form default.

8 Conclusion

A Stokes settling calculator is auditable when the velocity, the Reynolds number, and the drag law are the same statement. The Reynolds number uses the speed. Below 0.1 the statement is (2). Above 0.1, and while the Schiller–Naumann root stays below 1000, the statement is the unique positive root of (7), reached from the Stokes Reynolds number $K/24$ in a handful of Newton steps. Past that range the statement is $C_D = 0.44$, named as such. On the default $10\,\mu$m mineral sphere the two speeds differ by 0.472% and the residual is zero. On a $100\,\mu$m sphere the Stokes speed is 42.69% high. On a rigid sphere that rises at a Stokes velocity of $-0.436\,\mathrm{m\,s^{-1}}$, a signed Reynolds number of -872 is not evidence that the Stokes regime is valid. The drag-corrected Reynolds number is 152.09.

Data and code availability

The calculator is the AgentCalc page stokes-settling-velocity-calculator, and the released copy is in the agentcalc_science repository. The benchmark script `generate_stokes_settling_figure.py` and the series plotted in Figure 1 are archived with this note. No laboratory settling series was analyzed.

References

[1] George Gabriel Stokes. On the effect of the internal friction of fluids on the motion of pendulums. *Transactions of the Cambridge Philosophical Society*, 9:8–106, 1851.

[2] Roland Clift, John R. Grace, and Martin E. Weber. *Bubbles, Drops, and Particles.* Academic Press, New York, 1978.

[3] Osborne Reynolds. An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels. *Philosophical Transactions of the Royal Society of London*, 174:935–982, 1883. doi: 10.1098/rstl.1883.0029.

[4] L. Schiller and A. Naumann. Über die grundlegenden berechnungen bei der schwerkraftaufbereitung. *Zeitschrift des Vereines deutscher Ingenieure*, 77(12):318–320, 1933.

[5] Nian-Sheng Cheng. Comparison of formulas for drag coefficient and settling velocity of spherical particles. *Powder Technology*, 189(3):395–398, 2009. doi: 10.1016/j.powtec.2008.07.006.

[6] J. F. Richardson and W. N. Zaki. Sedimentation and fluidisation: Part I. *Transactions of the Institution of Chemical Engineers*, 32:35–53, 1954.